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ON THE NORM OF DECOMPOSABLE SUBGROUPS IN LOCALLY FINITE GROUPS

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We study the relationships between the norm of decomposable subgroups and the norm of Abelian noncyclic subgroups in the class of locally finite groups. We also describe some properties of periodic locally nilpotent groups in which the norm of decomposable subgroups is a non-Dedekind norm.

In the theory of groups, numerous results are connected with the investigation of the properties of groups with given restrictions imposed on their subgroups and systems of these subgroups. On the one hand, a group may have a system of subgroups with given properties but the influence of this system of subgroups is insignificant. On the other hand, the presence of even one (as a rule, characteristic) subgroup with a certain property can be a determining factor for the structure of the entire group. In recent years, the list of these subgroups can be noticeably enlarged by using various Σ-norms of the groups.

Recall that a Σ-norm of the group G is defined as the intersection of normalizers of all subgroups of the group contained in the system Σ. It is clear that, in the case where the Σ-norm coincides with the group, all subgroups of the group contained in Σ are normal (under the condition that the system Σ is nonempty). For the first time, groups with this property were considered by Dedekind at the end of the 19th century. He gave a complete description of finite groups in which each subgroup is normal (at present, they are called Dedekind groups). However, the systematic investigation of groups with arbitrary normal systems of subgroups has been continued only in the second half of the last century, which somewhat suspended the study of Σ-norms. At present, the structures of groups that coincide with their Σ-norm are known for many systems of subgroups Σ. Therefore, the problem of investigation of the properties of groups with proper Σ-norms seems to be quite natural.

For the first time, this problem was posed by Baer as early as in the 1930s (see, e.g., [1]) for the system Σ of all subgroups of a given group. He said that the corresponding Σ-norm is the norm of a group and denoted it by $N(G)$. It is clear that the norm $N(G)$ is contained in the other Σ-norms, which, in turn, can be regarded as its generalizations.

In the present paper, we consider one of these generalizations, namely, a *norm of decomposable subgroups* of a group. In view of the discussion presented above, we use this term for the intersection of normalizers of all decomposable subgroups of the group G and denote this norm by N_G^d. Note that a *decomposable* subgroup of the group G is defined as a subgroup that can be represented in the form of the direct product of two nontrivial factors [2].

It follows from the definition of the norm N_G^d that, for $N_G^d = G$, all decomposable subgroups of the group G are normal (provided that the system of these subgroups is nonempty). The non-Dedekind groups with this property were studied in [2] and called *di-groups*.

In the case where the group G does not contain decomposable subgroups, we assume that $N_G^d = G$. The following statement describes the structure of locally finite non-Abelian groups in which the system of decomposable subgroups is empty:

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Proposition 1 [2]. *A non-Abelian locally finite group without decomposable subgroups is a group of one of the following types:*

(i) a (finite or infinite) quaternion 2-group;

(ii) a Frobenius group $G = A \leftthreetimes B$, where A is a locally cyclic p-group, B is a cyclic q-group, p and q are prime numbers, and $(p-1, q) = q$.

A Frobenius group (see [3]) is defined as the semidirect product $G = A \leftthreetimes B$ of two nontrivial subgroups A and B, where

$$B \cap g^{-1}Bg = E$$

for any elements $g \in G \setminus B$ and

$$A \setminus E = G \setminus \bigcup_{g \in G} (g^{-1}Bg).$$

It is clear that a group G has decomposable subgroups if and only if it has decomposable Abelian subgroups. Therefore, it is reasonable to study the problem under the assumption of existence of some decomposable Abelian subgroups of the group. In many cases, decomposable Abelian subgroups are noncyclic. This enables us to assume that the norm of decomposable subgroups is directly connected with the norm of Abelian noncyclic subgroups N_G^A. According to [4], we use this notation for the intersection of normalizers of all Abelian noncyclic subgroups of the group G provided that a system of these subgroups in G is nonempty.

The aim of the present paper is to establish relationships between the norms N_G^d and N_G^A in a class of locally finite groups and to study the properties of locally nilpotent periodic groups in which the norm of decomposable subgroups is non-Dedekind. In the case where the norm N_G^A coincides with the group G, all Abelian noncyclic subgroups of the group are normal (provided that the group contains subgroups of this kind).

1. Relationships between the Norms of Abelian Noncyclic and Decomposable Subgroups in Locally Finite Groups

In this section, we consider groups under the condition that they contain at least one noncyclic Abelian subgroup. This restriction is connected with the definition of the norm of Abelian noncyclic subgroups.

In what follows, we need the following lemma:

Lemma 1.1. *Let G be a group containing a nonidentity N_G^d-admissible subgroup H such that $H \cap N_G^d = E$, where N_G^d is the norm of decomposable subgroups of the group G. Then the subgroup N_G^d is Dedekind.*

Proof. Since the subgroup H is N_G^d-admissible, we get

$$G_1 = HN_G^d = H \times N_G^d.$$

Let x be an arbitrary element of the norm N_G^d and $1 \neq h \in H$. Then

$$\langle x, h \rangle \lhd \langle h \rangle N_G^d, \qquad \langle x, h \rangle \bigcap N_G^d = \langle x \rangle \lhd N_G^d.$$

Hence, the norm N_G^d is Dedekind, Q.E.D.

The following statement describes the relationships between the norm N_G^d of decomposable subgroups and the norm N_G^A of Abelian noncyclic subgroups in a class of p-groups (p is a prime number):

Theorem 1.1. *In an arbitrary locally finite p-group G, the norms of Abelian noncyclic subgroups and decomposable subgroups coincide*: $N_G^A = N_G^d$.

The proof of the theorem is based on Lemmas 1.2–1.4 established in what follows.

Lemma 1.2. *In a class of finite p-groups, the norms of Abelian noncyclic subgroups N_G^A and decomposable subgroups N_G^d coincide.*

Proof. The statement of the lemma follows from the fact that each Abelian noncyclic subgroup in a finite p-group is an Abelian decomposable subgroup, and vice versa.

Lemma 1.3. *In an infinite p-group G, the norms N_G^A and N_G^d coincide if one of the following conditions is satisfied:*

(i) G does not contain quasicyclic subgroups;

(ii) G contains quasicyclic subgroups but none of them is a maximal Abelian subgroup;

(iii) among the maximal Abelian subgroups of the group G, there exists a quasicyclic subgroup normal in P.

Proof. (i). In this case, the sets of decomposable Abelian and Abelian noncyclic subgroups coincide. Hence, $N_G^A = N_G^d$.

(ii). Let P be a quasicyclic subgroup of the group G. By the condition, P is not a maximal Abelian subgroup. Hence, there exists a subgroup $\langle x \rangle$ of prime order such that the group $H = \langle x \rangle \times P$ is Abelian. Since H is an N_G^d-admissible subgroup, $H^p = P$ is also N_G^d-admissible. Therefore, all Abelian noncyclic subgroups of the group G are N_G^d-admissible and $N_G^A = N_G^d$.

(iii). Let G be a non-Abelian p-group and let P be a normal quasicyclic subgroup which is a maximal Abelian subgroup in G. In this case, we can show that G does not contain quasicyclic subgroups other than P. Indeed, if P_1 is a quasicyclic subgroup of the group G such that $P \neq P_1$, then the condition $[G : C_G(P)] < \infty$ implies that $P_1 \subseteq C_G(P)$. However, in this case, the subgroup $G_1 = P_1 \cdot P$ is Abelian, which contradicts the maximality of P. Therefore, P is a unique quasicyclic subgroup in G. Thus, the norm of Abelian noncyclic subgroups coincides with the intersection of normalizers of the reduced Abelian noncyclic subgroups each of which is decomposable and, hence, $N_G^A = N_G^d$.

The lemma is proved.

Corollary 1.1. *A non-Abelian locally finite p-group G contains a normal quasicyclic subgroup which is a maximal Abelian subgroup in G if and only if $p=2$ and $G = P\langle b\rangle$, where P is a quasicyclic 2-subgroup, $|b| \in \{2,4\}$, $b^2 \in P$,* and *$b^{-1}ab = a^{-1}$ for any element $a \in P$. Moreover, $N_G^A = N_G^d$.*

Proof. Let P be a quasicyclic subgroup normal in G which is a maximal Abelian subgroup of G. If $p \neq 2$, then, by Corollary 1.13 in [5], $P \subseteq Z(G)$. In this case, for any element $x \in G \setminus P$, the subgroup $\langle x, P\rangle$ is Abelian, which contradicts the condition. Hence, $p = 2$.

Since $P \lhd G$, we get

$$[G : C_G(P)] = 2.$$

The maximality of P yields $C_G(P) = P$. If G contains a single involution, then G is an infinite quaternion 2-group and $N_G^d = N_G^A = G$. Otherwise, there exists an involution $b \notin G \setminus P$ and $G = P \leftthreetimes \langle b\rangle$, where $b^{-1}ab = a^{-1}$ for any element $a \in P$. In this case,

$$N_G^d = N_G^A = \langle a_2, b\rangle,$$

where $a_2 \in P$ and $|a_2| = 4$. The converse statement is obvious.

The corollary is proved.

Further, we consider the case where the set of maximal Abelian subgroups of the p-group G contains quasicyclic subgroups but none of them is normal. Moreover, if G is a locally finite p-group, then, by Theorem 1.5 in [5], the group G does not satisfy the minimality condition for subgroups. In this case, G contains an infinite elementary Abelian subgroup and $N_G^A \subseteq N_G^d$.

Lemma 1.4. *If a locally finite p-group G contains a nonnormal quasicyclic subgroup which is a maximal Abelian subgroup in G, then $N_G^A = N_G^d$.*

Proof. Let P be a nonnormal quasicyclic subgroup which is a maximal Abelian subgroup of the group G. In view of the previous remark, we obtain $N_G^A \subseteq N_G^d$.

We prove that the inverse inclusion is true. To this end, it suffices to show that the subgroup P is N_G^d-admissible. If $N_G^d = E$, then $N_G^A = N_G^d = E$ and the statement is proved.

For $1 < \left|N_G^d\right| < \infty$, it follows from the condition $N_G^d \lhd G$ that $P \subseteq C_G\left(N_G^d\right)$. Since P is a maximal Abelian subgroup of the group G, $N_G^d \subset P$ and, hence, P is an N_G^d-admissible subgroup of G.

Let $\left|N_G^d\right| = \infty$ and let the norm N_G^d be non-Dedekind. Then it either does not contain decomposable subgroups or all decomposable subgroups in it are normal. By virtue of Proposition 1 and Lemma 2 in [2], N_G^d contains a quasicyclic subgroup P_1 characteristic in N_G^d. However, in this case,

$$P_1 \lhd G, \quad P_1 \neq P, \quad P \subset C_G(P_1),$$

and the subgroup $G_1 = P_1 \cdot P$ is Abelian, which is impossible by condition.

Now let N_G^d be an infinite Dedekind group. By the condition, P is a maximal Abelian subgroup and $P \neq G$. Thus, N_G^d does not satisfy the minimality condition for subgroups and, hence, the lower layer A of the norm N_G^d is an infinite elementary Abelian subgroup normal in G.

In the group $H = AP$, we consider subgroups $H_k = A\langle b_k \rangle$, $k = 1, 2, 3, \ldots$, where $P = \langle b_1, b_2, \ldots, b_n, \ldots \rangle$, $b_{n+1}^p = b_n$, $n = 1, 2, 3, \ldots$. By virtue of Lemma 1.9 in [5], the center $Z(H_k)$ of each subgroup H_k is infinite. Hence,

$$Z(H_k) \cap A = A_k, \quad |A_k| = \infty, \quad \text{and} \quad A_k = \langle a_1 \rangle \times \langle a_2 \rangle \times \ldots .$$

Without loss of generality, we can assume that $\langle b_k \rangle \cap \langle a_i, a_j \rangle = E$ for some $i \neq j$. Then, for any element $a \in A \subseteq N_G^d$, we get

$$[a, b_k] \in \langle a_i, b_k \rangle \cap \langle a_j, b_k \rangle = \langle b_k \rangle .$$

Therefore, $P \lhd H = AP$ and, by Corollary 1.15 in [5], $C_H(P)$ has a finite index in H. However, in this case, P is not a maximal Abelian subgroup of the group G. By using the already proved Lemma 1.4, we complete the proof of Theorem 1.1.

As shown above, the locally finite p-group G with infinite norm N_G^d does not contain a nonnormal quasicyclic subgroup P which is a maximal Abelian subgroup in G.

Corollary 1.2. *If a locally finite p-group G contains a nonnormal quasicyclic subgroup P which is a maximal Abelian subgroup in G, then $\left|N_G^d\right| < \infty$.*

The well-known Schmidt p-group without center [5, p. 72] for which $N_G^d = N_G^A = E$ may serve as an example of a group of this kind.

We now consider the relationships between the norms of decomposable and Abelian noncyclic subgroups in nonprimary locally finite groups.

Theorem 1.2. *Let G be a finite nonprimary group containing an Abelian noncyclic subgroup. Then the inclusion $N_G^A \supseteq N_G^d$ is true and, moreover, the case $N_G^A \neq N_G^d$* is possible.

Proof. In the analyzed group, the set of Abelian noncyclic subgroups is a subset of the set of Abelian decomposable subgroups. Hence, $N_G^A \supseteq N_G^d$. The following example shows that the indicated norms can be different and, hence, completes the proof of the theorem:

Example 1.1. Let $G = A \rtimes B$ be a finite Frobenius group in which A is an elementary Abelian group of order p^2 (p^2 is a prime number) and let B be a nonprimary subgroup, $(|B|, p) = 1$. It is known (see, e.g., [6]) that the center $Z(B) \neq E$. Hence, $N_G^A = G$ and N_G^d does not contain A.

A similar statement is also true for a class of infinite periodic locally nilpotent nonprimary groups.

Theorem 1.3. *For any infinite periodic locally nilpotent nonprimary group* G, *the inclusion* $N_G^A \supseteq N_G^d$ *is true and, in addition, the case* $N_G^A \neq N_G^d$ *is realized.*

Proof. Let F be a noncyclic Abelian subgroup of the group G. If the subgroup F is decomposable, then it is an N_G^d-admissible subgroup. If the subgroup F is indecomposable, then it is a quasicyclic p-group. Since the group G is locally nilpotent and nonprimary, there exists a subgroup $\langle b \rangle$ of prime order $q \neq p$ permutable with F. In this case, $F \times \langle b \rangle$ is an N_G^d-admissible subgroup and, hence, $\langle F, b \rangle^q = F$ is also an N_G^d-admissible subgroup. Therefore, $N_G^A \supseteq N_G^d$.

The following example confirms the existence of groups in which the norms of noncyclic and decomposable Abelian subgroups are different and, thus, completes the proof of the theorem:

Example 1.2. In the group

$$G = ((A \times \langle b \rangle) \leftthreetimes \langle c \rangle) \times \langle d \rangle,$$

where A is a quasicyclic p-group, $|b| = |c| = p$, $|d| = q$, $[b, c] = a \in A$, and $|a| = p$ (p and q are different prime numbers), the norm of Abelian noncyclic subgroups $N_G^A = G$ and, for the norm of decomposable subgroups, we get

$$N_G^d = A \times \langle d \rangle = Z(G) \neq N_G^A.$$

The examples presented in what follows show that, in a class of infinite locally finite and nonlocally nilpotent groups, the cases $N_G^A \neq N_G^d$ and $N_G^A \subset N_G^d$ or $N_G^A \supset N_G^d$ are possible.

Example 1.3. Let $G = A \leftthreetimes \langle b \rangle$ be a Frobenius group in which A is an infinite elementary Abelian 7-group, $|b| = 6$, and $b^{-1}ab = a^5$ for any element $a \in A$.

Since G is a Frobenius group and

$$N_G(\langle b \rangle) = \langle b \rangle, \quad N_G\left(\langle a^{-1}ba \rangle\right) = \langle a^{-1}ba \rangle, \quad \langle b \rangle \bigcap \langle a^{-1}ba \rangle = E$$

for $1 \neq a \in A$, we get $N_G^d = E$. On the other hand, $N_G^A = G$ and, hence, in this group, $N_G^A \supset N_G^d$.

Example 1.4 (see [6]). Let $G = A \leftthreetimes b$ be a Frobenius group in which A is an infinite elementary Abelian p-group $(p \neq 3)$ and let B be is a quasicyclic 3-group.

In this group $N_G^d = A$. Since

$$N_G(B) = B, \quad N_G\left(a^{-1}Ba\right) = a^{-1}Ba, \quad \text{and} \quad a^{-1}Ba \bigcap B = E$$

for $a \neq 1$, we get $N_G^A = E$. Hence, in this group, $N_G^A \subset N_G^d$.

Theorem 1.4. *In an arbitrary locally finite group* G *containing an Abelian noncyclic subgroup, either* $N_G^A = N_G^d$ *or* $N_G^A \supset N_G^d$, *or* $N_G^A \subset N_G^d$.

Proof. It suffices to show that it is impossible to find a locally finite group G for which

$$N_G^A \neq N_G^d, \quad N_G^A \not\supset N_G^d, \quad \text{and} \quad N_G^A \not\subset N_G^d.$$

Assume that this group G exists. Then, by virtue of Theorems 1.1 and 1.2, the group G is infinite and not primary. Moreover, it follows from the condition $N_G^A \neq N_G^d$ that this group contains an Abelian noncyclic subgroup P which is not N_G^d-admissible and a nonprimary cyclic subgroup $\langle b\rangle$ which is not N_G^A-admissible. It is clear that P is indecomposable and, hence, P is a quasicyclic group. We now show that P is a maximal Abelian subgroup of the group G. Indeed, otherwise, there exists a nonidentity subgroup $\langle g\rangle$ such that

$$P \cap \langle g\rangle = E.$$

Then the subgroup $P \times \langle g\rangle$ is N_G^d-admissible and, hence, the subgroup $\langle P, g\rangle^{|g|} = P$ is also N_G^d-admissible, which contradicts its choice. Thus, P is an Abelian group maximal in G.

Assume that $\left|N_G^d\right| = \infty$. Then

$$\left[\, G : C_G(N_G^d) \,\right] < \infty$$

and P belongs to the centralizer $C_G(N_G^d)$. However, this is impossible because the subgroup P is not N_G^d-admissible. Therefore, $\left|N_G^d\right| = \infty$.

The last remark implies that N_G^d contains an infinite Abelian subgroup M. Since $\langle b\rangle$ is nonprimary, we get

$$\langle b\rangle \lhd G_1 = \langle b\rangle M .$$

Thus,

$$\left[\, G_1 : C_{G_1}(b) \,\right] < \infty$$

and $C = C_{G_1}(b)$ is an infinite nonprimary Abelian group.

Assume that C does not satisfy the minimality condition for subgroups. Then this group contains noncyclic subgroups C_1 and C_2 such that

$$C_1 \cap \langle b\rangle = C_2 \cap \langle b\rangle = E.$$

In this case, the subgroups $C_i \times \langle b\rangle$, $i = 1, 2$, are N_G^A-admissible and, hence, the subgroup

$$\langle b\rangle = (\, C_1 \times \langle b\rangle\,) \cap (\, C_2 \times \langle b\rangle\,)$$

is also N_G^A-admissible, which is impossible by its choice.

Hence, C is a group with minimality condition for subgroups. However, in this case, the norm N_G^d also satisfies the minimality condition for subgroups and, according to the results obtained in [7], it is a finite extension of the complete subgroup $\tilde{P}$. By Corollary 1.3 in [5], the group $H = PN_G^d$ also satisfies the minimality condition for subgroups. Since P is a maximal Abelian subgroup of the group G, we conclude that $\tilde{P} = P$. This implies that P is a normal subgroup in H. The obtained contradiction proves that the analyzed case is impossible.

The theorem is proved.

2. Locally Nilpotent Periodic Groups with Non-Dedekind Norm of Decomposable Subgroups

In [2], it is shown that any non-Dedekind locally nilpotent periodic di-group containing at least one decomposable subgroup is a p-group in which all noncyclic Abelian subgroups are normal. The non-Hamiltonian groups with this property were studied in [8] and called $\overline{HA}$-groups. A similar statement is also true for the norm N_G^d of decomposable subgroups in the class of periodic locally nilpotent groups.

According to Theorem 1.1, the description of locally finite p-groups with non-Dedekind norm N_G^d of decomposable subgroups is reduced to the description of groups with non-Dedekind norm N_G^A of noncyclic Abelian subgroups. These groups were studied in [9, 10]. Based on these results, we easily verify that the following statements are true:

Lemma 2.1. *The norm N_G^d of a locally finite p-group G is non-Dedekind and does not contain decomposable subgroups if and only if $G = N_G^d$ and G is a (finite or infinite) quaternion 2-group of order greater than 8.*

Proof. The *sufficiency* of the conditions of the lemma follows from Proposition 1. We now prove their *necessity*.

Assume that G is a locally finite p-group and that its norm N_G^d is non-Dedekind and does not contain decomposable subgroups. Then, by virtue of Proposition 1, $p = 2$ and N_G^d is a (finite or infinite) quaternion 2-group. Moreover, $N_G^d = A\langle b\rangle$, $b^2 \in A$, $|b| = 4$, $|A| > 4$, A is a cyclic or quasicyclic 2-group, and $b^{-1}ab = a^{-1}$ for any element $a \in A$.

We now show that G contains one involution. Assume that this is not true and

$$x \in G \setminus N_G^d, \qquad |x| = 2.$$

Then $[x, b^2] = 1$, where b^2 is an involution of the norm N_G^d. Since the subgroup $\langle x, b^2\rangle$ is N_G^d-admissible, we have

$$\langle x, b^2\rangle \lhd G_1 = \langle x\rangle N_G^d \quad \text{and} \quad \left[\, G_1 : C_{G_1}(\langle x, b^2\rangle)\,\right] \le 2.$$

If $[x, b] \ne 1$, then $[x, b] = b^2$ and $|xb| = 2$. Then the Abelian subgroup $\langle xb, b^2\rangle$ is N_G^d-admissible, which is impossible because the element $a \in A$, $|a| = 8$, does not belong to the normalizer $N_G\left(\langle xb, b^2\rangle\right)$ of this subgroup. Hence, $[x, b] = 1$. Since $\langle x, b\rangle$ is a decomposable Abelian subgroup, it is N_G^d-admissible. However, in this case, the element $a \in A$, $|a| = 8$, also does not belong to the normalizer of the subgroup $\langle x, b\rangle$.

Thus, the group G contains only one involution and, hence, all Abelian subgroups of the group are indecomposable. By virtue of Proposition 1, G is a (finite or infinite) quaternion 2-group. By the condition, the norm N_G^d is non-Dedekind and, hence, $|G| > 8$ and $G = N_G^d$.

The lemma is proved.

Corollary 2.1. *A locally finite p-group G with a non-Dedekind norm N_G^d does not contain decomposable subgroups if and only if its norm N_G^d does not contain these subgroups.*

Lemma 2.2. *An infinite locally finite p-group G with a non-Dedekind norm N_G^d of decomposable subgroups is a finite extension of a quasicyclic subgroup.*

Proof. Let G be an infinite locally finite p-group and let N_G^d be its norm of decomposable subgroups. If the norm N_G^d does not contain decomposable subgroups, then, by Lemma 2.1, $G = N_G^d$ is an infinite quaternion 2-group. Let N_G^d contain a decomposable subgroup. By Theorem 1.1, $N_G^d = N_G^A$. Hence, G is an infinite locally finite p-group in which the norm N_G^A of Abelian noncyclic subgroups is a non-Hamiltonian $\overline{HA_p}$-group. By Corollary 4 in [10], G is a finite extension of a quasicyclic p-group, Q.E.D.

Theorem 2.1. *A periodic locally nilpotent group G containing a noncyclic Abelian subgroup has the non-Dedekind norm N_G^d of decomposable subgroups if and only if G is a locally finite p-group with non-Dedekind norm N_G^A of Abelian noncyclic subgroups.*

Proof. The *sufficiency* of the conditions of the theorem directly follows from Theorem 1.1.

We now prove their *necessity*. Let G be a periodic locally nilpotent group with non-Dedekind norm N_G^d of decomposable subgroups. Then N_G^d contains the non-Dedekind Sylow p-subgroup $(N_G^d)_p$ for a certain prime number p. By Lemma 1.1, $N_G^d = (N_G^d)_p$ and, moreover, G is also a p-group. By using Theorem 1.1, we conclude that $N_G^A = N_G^d$. Therefore, G is a p-group with non-Dedekind norm of noncyclic Abelian subgroups N_G^A.

The theorem is proved.

Corollary 2.2. *An arbitrary infinite periodic locally nilpotent group G with non-Dedekind norm N_G^d is a finite extension of a quasicyclic p-subgroup.*

Corollary 2.3. *If the norm N_G^d of a periodic locally nilpotent group G is infinite and non-Dedekind, then all noncyclic Abelian and all decomposable subgroups in G are also normal.*

Proof. The assertion of the corollary follows from Theorem 2.1 and Corollary 4 in [10].

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