РОЗДІЛ 1. АКТУАЛЬНІ ПИТАННЯ ПІДВИЩЕННЯ ЯКОСТІ НАВЧАННЯ ДИСЦИПЛІН ПРИРОДНИЧО-МАТЕМАТИЧНОГО ЦИКЛУ В ШКОЛІ ТА ВИЩИХ НАВЧАЛЬНИХ ЗАКЛАДАХ РІЗНИХ РІВНІВ АКРЕДИТАЦІЇ

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ABOUT THE MONITORING SYSTEM OF QUALITY OF PREPARATION TO EIA IN MATHEMATICS IN UKRAINE

In modern conditions, the relevance of the monitoring of the quality of the preparation for EIA does not raise doubts about the mathematics of senior school pupils. Particularly, very important is the timely diagnosis of existing gaps in the preparation of theoretical material and during solving test tasks of various forms during all the term of studying in grade 11. In this article, we present one of the possible models for organizing a system for monitoring the preparation for the external assessment in mathematics of Ukrainian graduates.

We will also develop methodological guidelines for the creation and implementation of diagnostic and training tests during preparation for the independent assessment of the quality of knowledge in mathematics. Special attention is paid to preparation for solving problems of open form with full explanation. In particular, we present here complete solutions, evaluation schemes and methodological comments for such tasks.

The given scheme of monitoring organization and the mentioned above methodological recommendations should help to ensure the proper quality of training of graduate pupils to the EIA in mathematics.

Keywords: *EIA in mathematics, SFA in mathematics, monitoring of preparation quality, educational achievements in mathematics, diagnostic tests, training tests, tasks with full explanation.*

Formulation of the problem. At present, the system of external independent assessment (EIA) of the quality of mathematical knowledge has finally been established in Ukraine as a means of conducting for state final attestation (STA) of graduates' educational achievements and the tool of competitive selection of applicants for higher education institutions. Therefore, the need for methodological investigations devoted to various aspects of preparation to the EIA in mathematics in modern conditions does not put into question. One of such aspect is monitoring the quality of graduates' preparation for testing during the 11th grade training.

It is clear that preparing for EIA in mathematics provides not only in the final class, but it is the period in which the process of systematization and repetition of the educational material reaches its peak. Therefore, timely diagnostics of the availability of gaps in the knowledge of theoretical material and the ability to solve the test tasks of various forms relating to the main content lines of the school course of math, is extremely important.

We suppose that the monitoring of the quality of the preparation to the EIA in mathematics will only be effective, if this diagnosis of gaps is carried out throughout all the school year. Therefore, it is extremely important to correctly organize a system of monitoring activities, in particular, to provide teachers by high-quality teaching materials for all stages of monitoring. In

addition, it is useful to establish a system of continuous communication between all participants in the process of preparing to the EIA: pupils, teachers, parents, administrators of different levels.

Analysis of actual research. The problem of preparing of pupils to the EIA and the STA in mathematics is systematically considered in the specialized scientific and pedagogical journals. V.G. Bevz, M.I. Burda, G.I. Bilyanin, O.Ya. Bilyanina, O.P. Vashulenko, L.P. Dvoretskaya, O.V. Yergina, O.S. Ister, A.G. Merzlyak, Ye.P. Nelin, V.B. Polonsky, V.K. Repeta, A.M. Roganin, O.P. Tomashchuk, M.S. Yakir and other are actively working in this direction and constantly publishing the results of their investigations.

Our author's team has been actively working over the last twelve years to provide methodological support for the process of preparation to the EIA in mathematics. The basics of the theory and methodology for assessment of academic achievements of pupils of senior school in Ukraine are described in the monograph [1]. We use methodical kit from the manuals [2] and [3] for the preparation of pupils to the EIA in mathematics.

The purpose of the article. The purpose of this article is to provide advice on organizing the monitoring of the quality of graduates' training in math during their studies in the 11th form and the methodical recommendations for solving math test items formulated in open form with full explanation.

Research methods. In order to achieve this goal, theoretical methods were used in the work: analysis of methodological literature on the research subject and empirical methods: observation of the training process of the pupils during their studying on training courses for the EIA in mathematics and analysis of the results of their achievements. The research also used a set of methods of scientific cognition: a comparative analysis to find out different views on the problem and determine the direction of research; systematization and generalization for the formulation of conclusions and recommendations for the preparation for nationwide standardized assessments of academic achievements in mathematics; generalization of author's pedagogical experience and observations.

Main material presentation. Diagnostic and training tests are the main tool for determining the availability and depth of pupils' problems during preparation to the independent assessment of the quality of knowledge. Naturally, the forms of used in them test items must be the same as in the test of external evaluation: multiple choice questions, short-answer tasks, structured short-answer tasks, matching tasks (tasks on logical pairs searching) and tasks of open forms with full explanation. It is also important that these tests carry out a proper thematic and typological coverage, that is, contain tasks that address to all the main topics and all the main types of exercises mastered by pupils during the EIA test.

It is clear that each diagnostic or training test can not cover all the topics and types of exercises and tasks mastered by pupils. Therefore, at each stage of the monitoring of the quality of preparation to the EIA in mathematics, several tests should be used. For example, in the beginning of the academic year, it is worth not giving one, but two or even three tests, which are conducted at intervals of several weeks. At the same time, each of these tests, covering the majority of the main themes and types of exercises and tasks, must nevertheless make a certain emphasis on this or that educational material. So, the first incoming diagnostic test, which is scheduled for the beginning of the school year in class 11, should have a proper thematic and typological coverage, but should be more focused, for example, on expression transformations, properties of functions and methods for solving equations, than on the properties of numerical inequalities and methods for solving inequalities with one variable. Obviously, the second incoming diagnostic test will necessarily include the task of applying the properties of numerical inequalities, solving inequalities by interval method, etc. The problems for transforming expressions, properties of functions and solving equations will be present in it in lesser quantities.

After mastering the material of class 11, but before the systematization and repetition, it is also advisable not to give one, but two or even three final diagnostic tests, which are carried out at intervals of several weeks. At the same time, each of these tests, covering the majority of the main themes and types of exercises and tasks, must nevertheless make a certain emphasis on this or that educational material. Natural in the final diagnostic tests is the proper thematic and typological

coverage with simultaneous concentration of attention on the newly studied topics: power and logarithmic function, derivative and its application, indefinite and definite integral, polyhedra, bodies of rotation, area of the bodies surfaces and volumes of the bodies.

In our opinion, the system for monitoring the quality of graduates' training to the EIA should contain a series of diagnostic and training tests that are natural to be carried out at the beginning of the school year (September), after studying the 11th grade material, but before the repetition (December-February) and immediately before testing (April-May). It is necessary to distinguish between diagnostic and training tests.

The main purpose of the *diagnostic tests* is to determine the quality of mastering of theoretical material and the quality of the graduates' ability to solve the exercises and tasks of various types and forms. These tests are not required to simulate a real EIA test in mathematics, have not a thematic structure similar to the one and the same number of tasks. The structure of the diagnostic test essentially depends on the amount of time that the teacher can take away. We suppose that it is an unnatural situation in which some administrative structures during the «monitoring» propose pupils to complete the test of the EIA format that contains 33 tasks, for 60 – 80 minutes.

We propose to conduct diagnostic tests lasting up to 80 minutes in the beginning of the school year and after studying the 11th grade material. Such test *can* contain 20 tasks: 12 tasks with multiple choice, 2 matching tasks, 4 tasks with a short answer, 2 of them structured, and 2 task of open form with full explanation. In this case, tasks 1-14 and 15-20 of this test separately carry the appropriate thematic coverage and can be used as separate diagnostic tests, calculated each for 40 minutes.

For each of multiple choice tasks 1-12 it is provided no more than 2 min, and for each of the matching tasks 13-14 – no more than 8 min. A teacher who works in a class where mathematics is taught at the standard level can be limited to tasks 1-14 of the test to diagnose an existing state of pupils' level of training to the EIA, since they do not go beyond the appropriate math program for such classes. This diagnostic test will calculated for 40 min (one lesson). For each task 15-18 with a short answer it is provided not more than 5 min, and to complete each of the tasks 19-20 with full explanation – no more than 10 min. A teacher who works in classes where mathematics is studied at the profile or advanced level can use either all tasks 1-20 (test duration 80 min, two lessons), or only tasks 15-20 (test duration 40 min, one lesson).

Thus, the purpose of the diagnostic test, first of all, is *educational*. This test is aimed for identifying of gaps in the training of the graduates to the IEA. The *training test* has a slightly different function. It is conducted directly before the testing (in April and May) and is intended to simulate the situation of the real test of the EIA, and therefore, in a certain sense, should «copy» this test with all the characteristics: by the number of tasks of different forms, in terms of complexity, by subject coverage, by time for performance, etc. Training tests should also be several and they are a separate component of monitoring the quality of preparation for external independent assessment.

During passing the final training tests it is necessary to create conditions for pupils that are as close as possible to the conditions of a real EIA test in mathematics. This refers to the time spent (180 minutes), the conditions for conducting (on their own, in a separate audience, without any assistance from the teacher, but under his control), the content of the test (it must fully comply with the program in mathematics and do not contain any additional tasks, beyond the boundaries of this program), as well as the organization of examination and evaluation of the test (preferably with the participation of the pupil). The graduate, however, must understand that the results of the training test are only a certain approximation of the possible outcomes of the test, and therefore should not underestimate or overestimate these results.

A teacher of mathematics during analysis of training tests should help the pupil to correctly treat their results. From this, how accurately, correctly and adequately this analysis will be conducted, depends the psychological state of the pupil before the test. Therefore, the teacher should think carefully about how to present the results of the training test, depending on the individual psychophysiological features of each particular pupil. A competent combination of

tactful indication of the mistakes made and consideration of similar examples will improve the pupil's results during the EIA in mathematics.

Make a few comments about *interpreting the results* of diagnostic and training tests. The main purpose of diagnostic tests is the study of those topics of the school course of mathematics and those types of exercises and tasks in which pupils have gaps. However, quite often pupils, their parents, and the administration are interested how it is possible to interpret the score for a diagnostic test in terms of real EIA test, that is, on a scale from 100 to 200 points. It is clear that using of last year scale to transfer test points to the 100-point scale, in general, is not correct. However, since in this situation the resulting score is only a benchmark for the subsequent preparation for testing, then this incorrectness can be neglected.

The maximum score for completing the EIA test in mathematics and in the training test is, as you know, is equal to 62 points. Therefore, you can use the training tests immediately, for example, in the 2018 scale, which is available at the link: https://osvita.ua/test/rez_zno/61141/. The maximum score of mentioned above diagnostic tests is 40 points, and the maximum score only for the tasks 1-14 or tasks 15-20 is 20 points. To convert the diagnostic test score to an analogue of the EIA test score, you can multiply the pupil's result by the conversion factor (it is equal to 1.55 for the entire test and 3.1 for each of its two parts), and then use the scale of the previous years to transfer the test scores to points from 100 to 200 and from 1 to 12.

It is also have to be explained for the pupils, their parents, and also for the school administration that every year a new scale of translation is being constructed, which depends on the complexity of the EIA test, as well as the results of test participants who perform this test. The details of the scaling process during standardized assessments of academic achievements are described in detail on the UCEQA website www.testportal.gov.ua.

Based on the author's experience, we note that the greatest difficulty in preparing for EIA in mathematics for senior school pupils is the solving and design of solutions of test items of open form with full explanation. Therefore, we will provide a solution to the tasks of this form, which could be a block of tasks with full explanation of some training test, as well as give full solutions and methodological comments to them.

31. Let $f(x)=\sqrt{2-x}$. 1) Find the set of function definition $D(f)$. 2) Draw the graph of $f(x)$. 3) Find the equation of tangent drawn up to the function $y=f(x)$ at the point where the abscissa is $x_0=-2$.

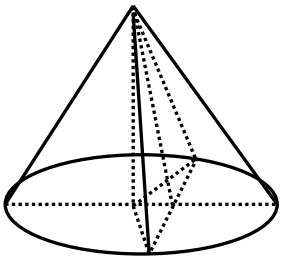
Solution. 1) The set of function definition consist of every number that satisfy the inequality $2-x\geq 0$. So, $D(f)=(-\infty;2]$. 2) For drawing of the function $f(x)$ graph we initially draw the graph of $y=\sqrt{x}$, then let's do its symmetry with respect to the ordinate axis and obtain the graph of $y=\sqrt{-x}$. After that we carry out the parallel transfer of the graph of the last function along the abscissa axis to 2 units to the right and get the graph of functions $y=\sqrt{-(x-2)}=\sqrt{2-x}=f(x)$. 3) The equation of tangent to function graph $f(x)$ in the point with abscissa x_0 has the form $y=f(x_0)+f'(x_0)(x-x_0)$. For our task $f(x_0)=\sqrt{2-(-2)}=2$, $f'(x)=\dfrac{-1}{2\sqrt{2-x}}$, $f'(x_0)=\dfrac{-1}{2\sqrt{2-(-2)}}=-0,25$. Thus, the required tangential equation is $y=2-0,25\cdot(x+2)$. After the transformations we have the following equation: $y=-0,25x+1,5$.

Comment. The assessment scheme for this task can be as follows. If the pupil correctly finds the set of function definition $f(x)$, then he will receive 1 point. Another point he gets if he has correctly plotted the graph of function $f(x)$. If the pupil correctly finds the derivative $f'(x)$, then he receives another 1 point. Finally, if the pupil correctly finds the equation of tangent, then he receives another 1 point. So, for the complete and correct solving of the whole task 31 pupil receives 4 points.

This task is not complicated, but requires precision when executed. It is not necessary to require students to give a detailed explanation of all stages of constructing the graph of function $f(x)$, but they need to focus their attention on the carefulness of the implementation of the final drawing, which must necessarily be marked the point $(2;0)$. Every stage of drawing of the tangential equation must be described in sufficient detail, not only the final answer have to be written down.

32. A cone with the height equals to 8 cm and the base radius equals to 6 cm is given. A cross section, which is an equilateral triangle, is made through the vertex of this cone. Find the angle α that forms the plane of this section with the plane of the cone base.

Solution. Let the picture depict this cone, point S is cone vertex, $SO = h$ is height, SAB is wanted section, $AO = OB = R$ are radiuses of base. Under the condition of the task $h = 8$ cm, $R = 6$ cm. From the triangle SOA by the Pythagorean theorem creating line $SA = l = 10$ cm. Since the triangle SAB is equilateral, than $AB = 10$ cm.



Let us plot in the plane of the cone base the segment $OM \perp AB$. Whereas OM is the projection of SM to the cone base, than by the theorem of three perpendiculars $SM \perp AB$, therefore, $\angle SMO = \alpha$ is wanted.

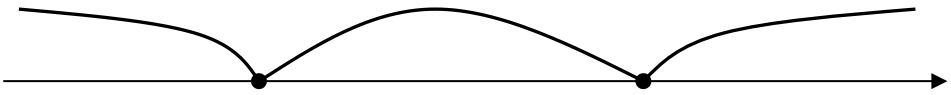
In the isosceles triangle AOB the height OM is also the median, wherefrom $AM = 5$ cm. From the rectangular triangle AOM by the Pythagorean theorem $OM = \sqrt{R^2 - AM^2} = \sqrt{11}$ cm. Thus, from the rectangular triangle SOM we have: $\operatorname{ctg}\alpha = \dfrac{OM}{h} = \dfrac{\sqrt{11}}{8}$ and $\alpha = \operatorname{arcctg}\dfrac{\sqrt{11}}{8}$.

Comment. The assessment scheme for this task could be the following. For the correct finding of the length of the creature pupil gets 1 point. Another 1 point he gets for the correct construction of the section in the figure with the indication of the angle between the planes. If the pupil correctly argued that the marked angle is the angle between the plane of the section and the plane of the base of the cone, he receives another 1 point. Finally, for the correct finding of the angle between the planes the student receives another 1 point. So, for the full and correct solution of the entire task 32 pupil receives 4 points.

During solving this problem it is necessary to focus the pupil's attention on the need to prove that $\angle SMO = \alpha$. In this case, it is necessary not only to refer to the theorem on the three perpendiculars, but also to clearly indicate the perpendicular, the slope and its projection. Note also that instead of finding $\operatorname{ctg}\alpha$ pupil can choose another trigonometric function of this angle, and if he finds it correctly, he will also receive the fourth score according to the assessment scheme. However, the fourth score will not be counted if in the answer will be written $\alpha = \operatorname{arcctg}\dfrac{\sqrt{11}}{8} + \pi n, n \in Z$, cause by this pupil will demonstrate a lack of understanding of the essence of the task putted to him.

33. Solve the equation $\log_3^2 x - 2a \cdot \log_3 x + a + 6 = 0$ for every values of the parameter a.

Solution. Perform the replacement $t = \log_3 x$. We have the equation $t^2 - 2a \cdot t + a + 6 = 0$. Let us find the discriminant: $D = 4a^2 - 4(a+6) = 4(a^2 - a - 6)$. By the F. Wiet theorem trinomial $a^2 - a - 6$ has the roots $a_1 = -2$ і $a_2 = 3$, thus, $D = 4(a+2)(a-3)$. Let us define the sign of the discriminant:



Therefore, for all $a \in (-2;3)$ $D < 0$, so, the quadratic and the initial equations have no roots. For all $a \in (-\infty;-2) \cup (3;+\infty)$ the quadratic equation has two roots

$t_{1,2}=\frac{2a\pm\sqrt{4(a^2-a-6)}}{2}=a\pm\sqrt{a^2-a-6}$ and the initial equation also has two roots $x_{1,2}=3^{a\pm\sqrt{a^2-a-6}}$. For $a=-2$ the quadratic equation has one root $t=-2$ and the initial equation also has one root $x=3^{-2}=\frac{1}{9}$. Finally, for $a=3$ the quadratic equation has one root $t=3$ and the initial equation also has one root $x=3^3=27$.

Answer: for all $a\in(-2;3)$ the equation has no roots; for $a=-2$ the equation has one root $x=\frac{1}{9}$; for $a=3$ the equation has one root $x=27$; for all $a\in(-\infty;-2)\cup(3;+\infty)$ the equation has two roots $x_{1,2}=3^{a\pm\sqrt{a^2-a-6}}$.

Comment. The assessment scheme for this task could be the following. If the pupil correctly completes the replacement and transferred to the square equation, he gets 1 point. Another 1 point pupil gets if he correctly found discriminant of the square equation. If the pupil correctly identifies the zeros and the sign of the discriminant, then he receives another 1 point. He receives another 1 point if he has specified at which values of the parameter the square equation relative to t has no roots. If the pupil correctly finds the analytic expression for the roots of the initial equation, he receives another 1 point. Finally, if the student considered the case when the initial equation has only one root and correctly found this root for both values of the parameter in which it is possible, then he receives another 1 point. So, for the complete and correct solution of the entire task 33 pupil receives 6 points.

During analyzing this task with pupils it is needed to pay pupils' attention to the fact that in a six-point task, even the smallest stages of the decision can be estimated, and therefore, it is not necessary to miss the description of «obvious» transformations and substantiations. On the other hand, for the given equation, the set of definition of the variable ($x>0$) does not affect the solution in any way and is therefore not evaluated. It is also important to emphasize the fact that for equations with the parameter, you need to consider all the possible values of the parameter and specify the solution (if it exists) for each of them. At the same time, write the general answer is not necessarily. It is in principle that there no of the valid values of the parameter was omitted.

Conclusions. We believe that well-organized monitoring of the quality of training to EIA in mathematics will make the process of preparation to this type of assessment much easier for teachers and pupils. Indeed, the diagnostic and training tests, which are the main means of monitoring, allow teachers «to keep hand on the pace» of the problems that pupils have during systematizing and repeating the school's mathematics course. It also allows pupils and their parents to determine whether their level of expectations matches the real level of knowledge and skills. Thus, by means of monitoring observation, it is much easier to avoid possible conflict situations and to increase the efficiency of communication between all participants in the educational process.

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Школьний О.В., Захарійченко Ю.О. Про систему моніторингу якості підготовки до ЗНО з математики в Україні.

У сучасних умовах актуальність моніторингу якості підготовки до ЗНО з математики учнів старшої школи сумнівів не викликає. Особливо важливою при цьому є вчасна діагностика наявних прогалин у засвоєнні теоретичного матеріалу та під час розв'язування тестових завдань різних форм протягом всього терміну навчання в 11 класі. У даній статті ми наводимо одну з можливих моделей організації систему моніторингу підготовки до ЗНО з математики українських випускників.

Нами також розроблемо методичні рекомендації щодо створення і проведення діагностичних та тренувальних тестів під час підготовки до незалежного оцінювання якості знань з математики. Особливу увагу при цьому нами приділено підготовці до розв'язування завдань відкритої форми з повним поясненням. Зокрема, ми наводимо тут повні розв'язання, схеми оцінювання та методичні коментарі до таких завдань.

Наведена схема організації моніторингу та згадані методичні рекомендації мають сприяти забезпеченню належної якості підготовки учнів випускних класів до ЗНО з математики.

Ключові слова: *ЗНО з математики, ДПА з математики, моніторинг якості підготовки, навчальні досягнення з математики, діагностичні тести, тренувальні тести, завдання з повним поясненням.*

Школьный А.В., Захарийченко Ю.А. О системе мониторинга качества подготовки к ВНО по математике в Украине.

В современных условиях актуальность мониторинга качества подготовки к ВНО по математике для учащихся старших классов сомнений не вызывает. Особенно важной при этом является своевременная диагностика имеющихся пробелов в усвоении теоретического материала и при решении тестовых заданий различных форм в течение всего срока обучения в 11 классе. В данной статье мы приводим одну из возможных моделей организации системы мониторинга подготовки к ВНО по математике украинских выпускников.

Нами также разработаны методические рекомендации по созданию и проведению диагностических и тренировочных тестов при подготовке к независимому оцениванию знаний по математике. Особое внимание при этом нами уделено подготовке к решению заданий открытой формы с полным объяснением. В частности, мы приводим здесь полные решения, схемы оценивания и методические комментарии к таким заданиям.

Приведенная схема организации мониторинга и упомянутые методические рекомендации должны способствовать обеспечению надлежащего качества подготовки учащихся выпускных классов к ВНО по математике.

Ключевые слова: *ВНО по математике, ГИА по математике, мониторинг качества подготовки, учебные достижения по математике, диагностические тесты, тренировочные тесты, задачи с полным объяснением.*